Theory of Plasma Contactors Used in the Ionosphere

D. E. Hastings*
Massachusetts Institute of Technology, Cambridge, Massachusetts

The use of plasma contactors has been proposed as a means of enhancing the current flow through an electrodynamic tether. A simple isothermal spherical model of the plasma cloud around a contactor is outlined for a plasma contactor which is biased positively with respect to the ambient plasma and hence collects electrons. It is shown that for significant current amplification to occur, the plasma cloud must be turbulent. The amount of current amplification is obtained as a function of the ion current through the contactor. For ion currents of several amperes amplification factors of 2–6 can be obtained for potential drops in the range 100–500 V. For smaller ion currents, much larger amplification factors can be obtained.

Nomenclature

A_0 = current collection area of physical collector
$\boldsymbol{B}$ = magnetic field vector
c_S = ion acoustic velocity
e = charge on an electron
$\boldsymbol{e}_r$ = unit vector in radial direction
$\boldsymbol{E}$ = electric field
f_i = ion distribution function
$\bar{f}_i$ = ionization fraction at the source boundary
I_c = ion current emitted from contactor
$I_c{}'$ = ion current crossing $r = r_c$
I_e = electron current collected at the physical collector
$I_e{}'$ = electron current crossing $r = r_c$
I_{total} = total current $(I_e + I_c)$
I_r^c = random electron current crossing $r = r_c$
I_r = random electron current
I_{r_0} = random electron current collected to area A_0
$\boldsymbol{j}$ = current density
L_{n_i} = density scale length
m_e = electron mass
m_i = ion mass
n_e = electron density
n_e^{amb} = ambient electron density
n_i = ion density
n_r = neutral density
p_e = electron pressure
r_c = cloud radius at which $\nu_e/\Omega_e = 1$
r_0 = radius of physical collector
S_{ion} = ion creation rate due to ionization
S_{recom} = ion recombination rate
T_e = electron temperature
T_e^{amb} = ambient electron temperature
T_i = ion temperature
$\boldsymbol{V}_0$ = velocity of source measured from a frame fixed to the earth
$\boldsymbol{v}_e$ = electron drift velocity
$\boldsymbol{v}_i$ = ion drift velocity
v_{i_r} = radial ion speed
v_n = radial neutral velocity
v_{r_0} = ion radial velocity at $r = r_0$
w = turbulent energy
α = $\nu_{\text{eff}}^e/\omega_{p_e}$
λ_i = ion mean free path
ϕ = electrostatic potential
ϕ_0 = electrostatic potential at $r = r_0$
Ω_e = electron cyclotron frequency
Ω_i = ion cyclotron frequency
$\langle \sigma v \rangle_{\text{ion}}$ = ionization cross section ($\times$ velocity)
$\langle \sigma v \rangle_{\text{recom}}$ = recombination cross section ($\times$ velocity)
ν_e = $\nu_{ei} + \nu_{\text{eff}}^e$
ν_{ei} = electron ion collision frequency
ν_{eff}^e = anomalous electron collision frequency due to turbulence
ν_i = $\nu_{ie} + \nu_{\text{eff}}^i$
ν_{ie} = ion electron collision frequency
ν_{eff}^i = anomalous ion collision frequency due to turbulence
ω_{p_e} = electron plasma frequency

I. Introduction

THE electrodynamic tethered satellite system has been proposed[1] as a means of generating power in space. Power can be generated if a current can be made to flow through the ionosphere/tether combination. The simplest concept to achieve this is to put a large conducting balloon on the free end of the tether, collect electrons from the ionosphere to the balloon, have them flow through the tether and then be ejected out of the other end. For a useful generating device, power levels of 100 kW are desirable. This requires current levels in the range of 1–30 A. The random current density available in the ionosphere is approximately 1 mA/m^2 so to collect even 1 A of current would require a conducting balloon whose radius is 12.6 m. These large-size collectors are not considered desirable. Another way to achieve the same level of current is to use a plasma contactor. A plasma contactor is a plasma source which forms a large plasma cloud around the current collector. The descriptive name "plasma contactor" then describes the expectation that the plasma cloud volume will form an effective current collection area much larger than the physical area of the collector. Hence if all the current that enters the large cloud is channeled to the physical collector then the plasma contactor will have served to amplify the current flow relative to what would have been collected with just the physical collector.

The collection of current in the ionosphere has been considered previously for understanding satellite neutralization during electron beam experiments. When an electron beam is fired from a satellite, charge conservation requires that in steady state the net current to the satellite surface be zero. Hence a current of the order of the beam current (the return

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*Assistant Professor of Aeronautics and Astronautics. Member AIAA.

current) must be attracted back to the satellite. The satellite potential becomes positive in order to attract the return current to the surface if the return current exceeds the random thermal current. In 1961, Beard and Johnson[2] considered this problem and solved it for spherical-space-charge limited flow to a collector. Their model indicates that to collect current levels of amperes would require that the collector be biased to a several kilovolt potential. Such large biases are undesirable for building a viable system where most of the potential drop should appear across the load. Collector biases of the order of kilovolts would remove too much of the available potential drop (for a 100-km tether, this is approximately 20 kV). Since in Ref. 2 the fundamental effect of the Earth's magnetic field was neglected, the result of Beard and Johnson must be considered to be a lower limit to the potential of the collector. The Earth's magnetic field has a major effect on the collection of current. This is because most of the current is electrons (since their mass is so small compared to the ions) and the ambient electron gyroradius (energy $\simeq$ 0.1 eV) in the Earth's magnetic field is about 3 cm, which is small compared to the typical collector radius. Since the electron gyroradius is a measure of how far an electron can move across the magnetic field the current collection area is limited to approximately the area that the collector projects perpendicular to the magnetic field. This problem was considered in Ref. 3 where they derived an expression for the potential on a collector to collect a given current in the ionosphere. From their results it can be shown that ampere current levels would require potentials of the order of 5–10 kV, a figure larger than the Beard and Johnson result, as anticipated. However, it should be noted that, due to induced turbulence and additional ionization, spacecraft potentials during electron beam experiments do not reach near the space charge limited value.[4] Hence the theoretical predictions motivate the investigation of plasma contactors for current enhancement with tethers. Additional evidence for the efficacy of contactors comes from Shuttle experiments. This is because injection of a plasma cloud around a vehicle in space has been shown to be an effective means of clamping the vehicle potential to near the space potential.[5] Hence the potential bias on the collector can be substantially reduced by a plasma contactor.

In this report we develop a simple model for use of a plasma contactor in the ionosphere which collects electrons. We show that in order to get any current enhancement the plasma flow leaving the collector must have collisional electrons. In order to get useful current levels the plasma cloud must be highly turbulent. We obtain the potential drop in the cloud as a function of the total current flowing through the cloud. We show that typically total currents of 2–6 times the ion current for ion currents in the ampere range can be obtained for potential drops in the range 100–500 V and for turbulence levels which are high but perhaps not unobtainable. In Sec. II we develop the basic plasma model while in Sec. III we present the numerical results from the model.

II. A Simple Model for a Plasma Contactor Cloud

We shall follow the model of Refs. 6 and 7 and model the plasma cloud as being emitted from a spherical source of radius r_0 (Fig. 1). The source emits plasma cloud consisting of an ion current I_c plus neutralizing electrons and acts as an anode for collecting the electron current, I_e. We assume that none of the ions return to the collector hence the total current crossing the collector face is

$$I_{\text{total}} = I_c + I_e \tag{1}$$

We assume that the electrons near the source experience a sufficient level of collisions (both classical and turbulent) that they are not trapped on their gyro-orbits. If this assumption is not true then the electrons will be constrained to flow mainly along the field lines and as in the analysis of Ref. 3 the current collection area will be

$$A_0 = 2\pi r_0^2 \tag{2}$$

The factor of 2 occurs in (2) because current can be collected from both directions along the magnetic field. The random electron current which will cross an area A is

$$I_r = en_e \left(\frac{T_e}{2\pi m_e}\right)^{1/2} A \tag{3}$$

The ambient ionospheric electron density is $n_e^{\text{amb}} \simeq 1\times 10^5$ cm^{-3}, the ambient electron temperature $T_e^{\text{amb}} \simeq 0.1$ eV and for a collector radius $r_0 = 10$ cm, a random electron current I_{r_0} of 5.33×10^{-5} A can be collected from the ionosphere. Hence for currents in the ampere range the electrons must be collisional when they leave the source.

The two possible directions of anisotropy in the contactor system are the magnetic field direction (***B***) and the direction of motion of the source (V_0) (Fig. 1). For uniform motion and for dense plasma clouds ($n_e \geq 10^4$cm^{-3}) the plasma emitted from the source will not be affected by the motion of the source. This is because the equations of motion of the plasma particles are invariant under transformation to a uniformly moving frame with the exception that the electric field seen in a moving frame is different from a stationary frame of reference. However, for a dense cloud the motional electric field will be shielded out due to the formation of polarization currents. The only other way the motion of the source can be communicated to the cloud is for the incoming neutral wind to exchange momentum with the cloud therefore blowing it away antiparallel to the direction of motion. For a dense cloud this effect will be very small and we will neglect it. Since the electrons must not be completing their gyro-orbits on leaving the source and since by conservation of momentum the ions must not be completing their gyro-orbits, the plasma cloud will not be affected by the magnetic field direction when it leaves the source. Hence the plasma cloud will expand isotropically across the magnetic field until the density drops enough so that the electrons can complete their gyro-orbits. At that point the cloud will start to expand anisotropically with most of the expansion being along the magnetic field. However, for current collection once the cloud radius reaches the point where electrons complete their gyro-orbits, current can effectively only be collected along the magnetic field. If this critical radius is r_c then the current collection area is $A_c = 2\pi r_c^2$ and the random electron current from (2) is $I_r = I_{r_0}(r_c/r_0)^2$. Hence the plasma contactor will have enhanced the current collection ability of the collector of radius r_0 by $(r_c/r_0)^2$. In order to obtain this critical radius we need to solve the equations for the spherical expansion of a plasma cloud with collisional electrons.

We start with electron momentum balance in the frame of the source (ignoring electron inertia, the Lorentz force since the electrons are collisional and momentum transfer from

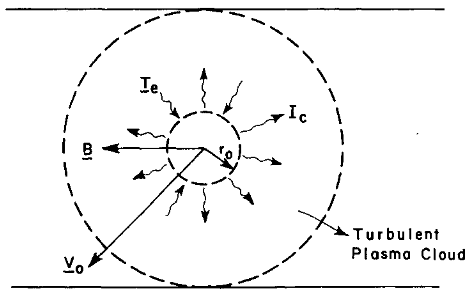

Fig. 1 Plasma source, radius r_0 moving across magnetic field, emitting a neutralized ion current I_c.

the ambient neutral wind)

$$0 = -\nabla p_e - en_e E - m_e n_e \nu_{ei}(v_e - v_i) - m_e n_e \nu^e_{\text{eff}}(v_e - v_i) \tag{4}$$

Since in plasmas turbulence is due to fluctuating electromagnetic fields it can be modeled as an effective collision frequency akin to the Coulomb collision frequency. In (4), we have assumed that the turbulence is current-driven since we take its momentum transfer as proportional to current density. We have neglected momentum transfer to neutrals ejected with the plasma cloud. This is because even for small ionization fractions, the Coulomb cross section for momentum exchange is so much larger than the electron neutral collision cross section that electron neutral collisions can be neglected. Typically plasma sources are expected to put out gas/plasma mixtures which are 1%-100% ionized. The current density in the plasma is $j = en_e(v_i - v_e)$ and in a spherically symmetric situation (as we have assumed here) we can write

$$j = j_r e_r = \frac{I}{4\pi r^2} e_r \tag{5}$$

where e_r is the unit vector in the radial direction. We can now write (4) in the radial direction and obtain

$$\frac{\partial \phi}{\partial r} = \frac{1}{en_e}\left(\frac{\partial p_e}{\partial r} - \frac{m_e \nu_e}{e}\left(\frac{I}{4\pi r^2}\right)\right) \tag{6}$$

From (6) we can obtain the potential drop across the plasma cloud once we know the radial variation of the electron density and temperature. The first term on the right-hand side of (6) is the familiar Boltzmann term and leads to the result that the potential has a component which varies with the density. The second term is the contribution to ϕ which leads to Ohm's law, that is the current is proportional to the applied field.

The plasma density can be obtained from the ion continuity equation. In steady state for spherical symmetry this is

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 n_i v_{i_r}) = S_{\text{ion}} - S_{\text{recom}} \tag{7}$$

The electron density is then given in the cloud by quasineutrality

$$n_e = n_i \tag{8}$$

The electron density will satisfy (7) and (8) until the density drops to the point where $\nu_e/\Omega_e = 1$. Here Ω_e is the electron cyclotron frequency given by $\Omega_e = eB/m_e$, where $B \simeq 0.45 \times 10^{-4} T$ for low Earth equatorial orbit. From momentum conservation it can be shown that at the same radial location the ions will satisfy $\nu_i/\Omega_i = 1$, where $\nu_i = \nu_{ie} + \nu^i_{\text{eff}}$. This is because in both a Coulomb and turbulent momentum exchange we must have $m_i\nu_{ie} = m_e\nu_{ei}$ and $m_i\nu^i_{\text{eff}} = m_e\nu^e_{\text{eff}}$. Thus we have

$$\frac{\nu_e}{\Omega_e} = \frac{m_e\nu_e}{m_e\Omega_e} = \frac{m_i\nu_i}{m_i\Omega_i} = \frac{\nu_i}{\Omega_i}$$

Hence in the plasma contactor cloud where $\nu_e/\Omega_e \geq 1$ we will have $\nu_i/\Omega_i \geq 1$. The ion distribution function satisfies in steady state

$$v \cdot \nabla f_i + \frac{e}{m_i}(E + v \times B) \cdot \frac{\partial f_i}{\partial v} = C(f_i) \tag{9}$$

where $C(f_i)$ is the ion collision operator. Since in the plasma cloud $\nu_i > \Omega_i$ we can expand (9) in the small parameter Ω_i/ν_i. We write

$$f_i = f_{i_0} + f_{i_1} + \ldots$$

where $f_{i_1} = 0((\Omega_i/\nu_i)f_{i_0})$. Then (9) becomes

$$v \cdot \nabla f_{i_0} + \frac{e}{m} E \cdot \frac{\partial f_{i_0}}{\partial v} = C(f_{i_0}) \tag{10}$$

In the plasma cloud we have shown that the ions are collisional with respect to the gyro-orbits. However, the ion gyro-orbit is so large (~6 m for low Earth orbit) and since the length scale of the plasma cloud is expected to be in the meter range the ions can be collisionless with respect to the scale length of the density. That is the ion mean free path can exceed the density scale length. A simple argument can show that even if this is not true close to the physical collector it will probably be true out in the plasma cloud. The ion mean free path λ_i is given by

$$\lambda_i \propto \frac{1}{n_i}$$

If $n_i \propto r^{-k}$ [(for example for $S_{\text{ion}} = S_{\text{recom}} = 0$ and for v_{i_r} = constant the solution to (7) is $n_i \propto r^{-2} \Rightarrow k = 2$)] then

$$\lambda_i \propto r^k$$

but $L_{n_i} = 1/n_i \cdot \partial n_i/\partial r^{-1} = r/k$ = density scale length. Hence for $k > 1$, λ_i increases faster with r than L_{n_i} and will probably exceed L_{n_i} at some radial location in the cloud. We assume that the ions are sufficiently collisionless with respect to the density variation that we can expand (10) in the parameter L_{n_i}/λ_i. The equation for the distribution function is then

$$v \cdot \nabla f_{i_{00}} + \frac{e}{m} E \cdot \frac{\partial f_{i_{00}}}{\partial v} = 0 \tag{11}$$

where $f_{i_{00}}$ is the zeroth order in the small parameter L_{n_i}/λ_i. The general solution is that $f_{i_{00}}$ is an arbitrary function of the constants of the motion. The functional form of the distribution function will then be determined by the distribution function for the ions at the source surface. For ions moving in a central forcefield the constants of the motion are the energy ϵ and the angular momentum J. For simplicity we shall consider a case where the ions are all ejected monoenergetically (energy ϵ_0) and radially from the source. The ion distribution function will be a delta function and the ion radial velocity at any radial location will be given by

$$\tfrac{1}{2} m_i v_{i_r}^2 + e\phi = \tfrac{1}{2} m_i v_{r_0}^2 + e\phi_0 = \epsilon_0 \tag{12}$$

where ϕ_0 is the potential at $r = r_0$ and V_{r_0} is the radial velocity with which all the ions are ejected. This assumption for the ions is based on the experimental observation of plasma sources which indicate that the potential drops are typically much larger than the ion thermal energy and so the ions move mainly in response to the radial potential. Another experimental observation[8] is that the ion flow is choked at the source boundary, that is the ion initial velocity is the Bohm or ion acoustic velocity. Therefore for this work we take

$$v_{r_0} = c_S = \sqrt{2T_e/m_i} \tag{13}$$

Since we wish to elucidate the basic phenomena and since it is the density variation which is crucial in determining the current collection radius we shall take the cloud to be isothermal. This is a fair assumption since transport of energy is much more rapid than transport of mass in plasma clouds. However, it leaves out the important phenomena due to ohmic heating of the cloud by the current. Consideration of these effects will be deferred for future work.

Since we included ionization and recombination in (7) we must have an equation for the neutral density n_n. This is taken to be

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 n_n v_n) = S_{\rm recom} - S_{\rm ion} \qquad (14)$$

where the neutrals are taken to expand collisionlessly with a velocity v_n given by

$$v_n \simeq \sqrt{2T_i/m_i}$$

The set of equations (6)–(8), (12), and (14) form a closed set of equations for n_i, n_n, v_{i_r}, n_e, and ϕ. The boundary conditions at $r=r_0$ can be taken to be

$$\phi=\phi_0$$

$$n_i = I_c/(4\pi r_0 e c_s)$$

$$n_n = n_i(1-\bar{f}_i)/\bar{f}_i$$

At $r=r_c$, where $\nu_e/\Omega_e=1$ the total current crossing the cloud boundary heading towards the physical collector is

$$I_{\rm total} = I_c' + I_e' \qquad (15)$$

where the prime indicates that because ions and electrons can be created or destroyed in the cloud the ion and electron currents at $r=r_c$ do not have to be the same as at $r=r_0$. However, steady-state charge conservation implies that the total current must be conserved. We can rewrite (15) as

$$I_{\rm total} = I_c + (I_c' - I_c) + I_e' = I_c + I_e$$

Hence the electron current at $r=r_c$ is related to the electron current at $r=r_0$ by

$$I_e' = I_e + (I_c - I_c') \qquad (16)$$

The electron current at $r=r_c$ must in steady state be bounded above by the random electron current which can cross $r=r_c$. Hence from (3) and (16) we have

$$I_e' \le e n_e^{\rm amb}\left(\frac{T_e^{\rm amb}}{2\pi m_e}\right)^{1/2} 2\pi r_c^2 = I_r^c \qquad (17)$$

Note that the random current is the current which crosses an area in the absence of any potential bias. Hence if $I_e' < I_r^c$ then there must exist a potential in the far field which repels just enough of the incoming electrons so that the current crossing r_c is I_e'. To complete (16) the ion current crossing r_c is

$$I_c = 4\pi r_c^2 e n_i v_i (r=r_c) \qquad (18)$$

The set of equations for densities and potential will be complete once $S_{\rm ion}$, $S_{\rm recom}$, and ν_e are given. We take $S_{\rm ion}$, $S_{\rm recom}$ as

$$S_{\rm ion} = n_e n_n \langle \sigma v\rangle_{\rm ion} \qquad (19)$$

$$S_{\rm recom} = n_e n_i \langle \sigma v\rangle_{\rm recom} \qquad (20)$$

where we take the ionization rate from Ref. 9 and the recombination rates from Ref. 10.

The electron collision frequency is taken as $\nu_e = \nu_{ei} + \nu_{\rm eff}^e$ with ν_{ei} from Ref. 11 and given by

$$\nu_{ei} = \frac{3.9\times 10^{-6}}{T_e^{3/2}} n_i \left(23 - \ln\left(\frac{n_e^{1/2}}{T_e^{3/2}}\right)\right) {\rm s}^{-1} \qquad (21)$$

where n_i and n_e are in cm^{-3} and T_e is in electron volts. The effective scattering frequency is chosen to be given by

$$\nu_{\rm eff}^e = \alpha\omega_{pe} = \alpha\left(\frac{4\pi n_e e^2}{m_e}\right)^{1/2} = \alpha 5.64\times 10^4 n_e^{1/2} {\rm s}^{-1} \qquad (22)$$

where n_e is in cm^{-3}. The choice of this model for $\nu_{\rm eff}^e$ is motivated by two observations. On theoretical grounds it can be shown that in a stable plasma[12] $\nu_{\rm eff}^e \propto \omega_{pe}$ and in a plasma unstable to ion acoustic waves the effective collision frequency is given by[13]

$$\nu_{\rm eff}^e \simeq \omega_{pe}\left(\frac{W}{nT_e}\right)$$

Hence in (24) we would expect the free parameter α to be related to the ratio of turbulent energy to thermal energy. Secondly, experimental results[14] indicate that anomalous resistivity is often found to be proportional to $\sqrt{n_e}$ as in (22). If the plasma cloud were unstable to ion acoustic waves and there were no other instability present then the parameter α in (22) can be evaluated theoretically and takes values in the range 1×10^{-3}–1×10^{-2}. This corresponds to the physically reasonable notion that the instability saturates as soon as the wave amplitude grows big enough for quasilinear effects or resonance broadening to affect the electrostatic wave. The turbulent energy required for these processes to set in is not very large, typically being less than 1% of the thermal energy of the plasma. On the other hand recent observations of the plasma cloud about the Space Shuttle have measured fluctuating densities[15] $\delta n_e/n_e \sim 0.1$–0.5. Since it is reasonable to assume $W/nT_e \propto (\delta n_e)^2/n_e^2$ then values of α in the range up to 0.25 may be possible. A further observation that may determine the value of α is that near the walls of plasma experiments[16] very often $\delta n_e/n_e \sim 1$ while well away from the walls $\delta n_e \ll n_e$. In view of the considerable uncertainty as to what sort of instabilities may occur in a contactor cloud and what the effect of the physical collector might be we shall take α to be a free parameter and determine what values are necessary to obtain a viable plasma contactor. We can make an estimate of this by the following calculation. If we ignore the electron ion collisions then the requirement $\nu_e = \Omega_e$ at $r=r_c$ becomes from (22)

$$\alpha 5.64\times 10^4 n_e^{1/2} \simeq \Omega_e \simeq 7.92\times 10^6$$

hence at r_c we have

$$n_e(r=r_c) \simeq \frac{1.97\times 10^4}{\alpha^2}\ {\rm cm}^{-3}$$

If we ignore ionization and recombination and take a constant ion velocity we have

$$n \simeq \frac{I_c}{4\pi r_0^2 e c_S}\frac{r_0^2}{r^2}$$

hence

$$r_c^2 \simeq \frac{I_c}{4\pi r_0^2 e c_S}\frac{r_0^2}{n_e(r=r_c)}$$

The electron current is bounded by the random electron current through $A_c = 2\pi r_c^2$ hence

$$I_e \le e n_e^{\rm amb}\left(\frac{T_e^{\rm amb}}{2\pi m_e}\right)^{1/2}\frac{I_c}{2ec_S}\frac{1}{n_e(r=r_c)}$$

$$\simeq 81 I_c \alpha^2$$

where we took $T_e = T_e^{\rm amb} = 0.1$ eV, $n_e^{\rm amb} = 10^5 {\rm cm}^{-3}$ and the ions as argon. This crude calculation indicates that for $I_e = I_c$

(i.e., $I=2I_c$ so the contactor has a gain of 2) we require $\alpha \gtrsim 0.11$. This figure suggests that for significant current amplification to occur the plasma cloud must have a high degree of turbulence.

III. Numerical Solution of the Plasma Model

The equations for n_i, n_n, n_e, v_{i_r}, and ϕ have been solved numerically by using a standard package for integrating first order equations (LSODE). The equations were integrated until $\nu_e/\Omega_e=1$. The ion species was chosen to be argon which has a mass of 40 proton masses and an ionization energy of 15.6 eV. The ionization fraction $\bar{f}_i$ was chosen to be 0.999 since it is clear that the biggest cloud will be obtained when the collisionality is maximized. This will be obtained when the plasma is fully ionized. The electron and ion temperatures were taken as 0.1 eV which is typically the energies at which plasma sources emit plasma. We show in Table 1 that variations in T_e produce little variation in potential drop. The source radius r_0 was chosen to be 10 cm. This is a reasonable radius at which plasma sources will start to manifest spherical plasma expansion if for example a 2-cm hollow cathode[17] is used as a plasma source.

In Fig. 2 the potential drop $\Delta\phi=\phi_0-\phi(r=r_c)$ across the cloud is shown vs the total current I for $I_c=0.01$ A. It was found that to get currents significantly larger than I_c required $\alpha \gtrsim 0.2$. Hence only the voltage-current relationships for $\alpha=0.2$ and $\alpha=0.4$ are shown. The voltage-current curve in each case is plotted to the point where the incoming electron current is just equal to the random electron current ($I_e'=I_r^c$). Beyond this current level a significant potential would have to be applied at the critical radius to attract electrons across the magnetic field. This would take potentials in the kilovolt range which are not acceptable for a viable system. For $\alpha=0.2$, a gain (I/I_c) of up to three can be obtained for a potential drop of a few volts. This clearly shows the effectiveness of a plasma contactor. For $\alpha=0.4$, a gain of up to 11 can be shown for a potential drop of 27 V. In Fig. 3, the critical radius r_c of the plasma cloud is shown as a function of the total current. The critical radius is seen to decrease as the current increases. This occurs because the resistive contribution to the potential increases with current so increasing the potential drop as the current increases. As the potential drop increases the ions are accelerated more and hence by continuity the density decreases more quickly. Hence the critical density is achieved at a smaller radius as the current increases. For $\alpha=0.4$ the critical radius for $I=0.11$ A reaches 4.3 m. This is a large cloud and indicates that the initial radius r_0 is not significant as long as it is much smaller than the critical radius. In Fig. 4, the voltage current curves for $I_c=0.1$, 1, and 10 A and for $\alpha=0.2$ and 0.4 are presented.

For $\alpha=0.4$ and $I_c=0.1$ A we see that a maximum gain of about 6½ can be obtained for a potential drop of 85 V. For the same value of α and $I_c=1$ A the maximum gain is about 4.2 for a potential drop of 270 V. This decrease in gain and increase in potential drop with increasing ion current occurs for the same reason, namely, the critical radius decreases with current. Larger currents mean larger potential drops which implies faster density drops. Hence the rate at which the critical radius increases with current is a decreasing function of current. From Fig. 4 we also see that while increasing the effective turbulence level increases the potential drop a much more important effect is that it allows the plasma cloud to get much larger and hence allows more total current. This leads us to the opposite conclusion from the conventional wisdom. It is usually thought that anomalous resistivity is a harmful phenomenon since it inhibits current flow. Here, since the very effectiveness of the plasma contactor depends on turbulent enhancement of the collision frequency anomalous resistivity is necessary to achieve the large cloud radii required for useful current amplifications. For $I_c=10$ A the maximum gain that can be achieved is about 2¾ for a potential drop of 900 V. This confirms the trends noted in Figs. 2 and 3. The critical radius for these high currents is of the order of 60 m, which is a very large plasma cloud. However, we note that for such clouds the assumption of collisionless ions is probably not correct. It was

Table 1 Maximum critical radius, total current, and potential drop for a plasma contactor emitting 10 A of neutralized ion current with an effective turbulence level $\alpha=0.4$ for several plasma temperatures

$T=T_e=T_i$, ev	I_c, A	Max r_c, m	I, A	$\phi_0-\phi(r=r_c)$, V
0.1	10	57.4	27.6	891
0.5	10	57.3	27.5	899
1.0	10	57.1	27.4	907

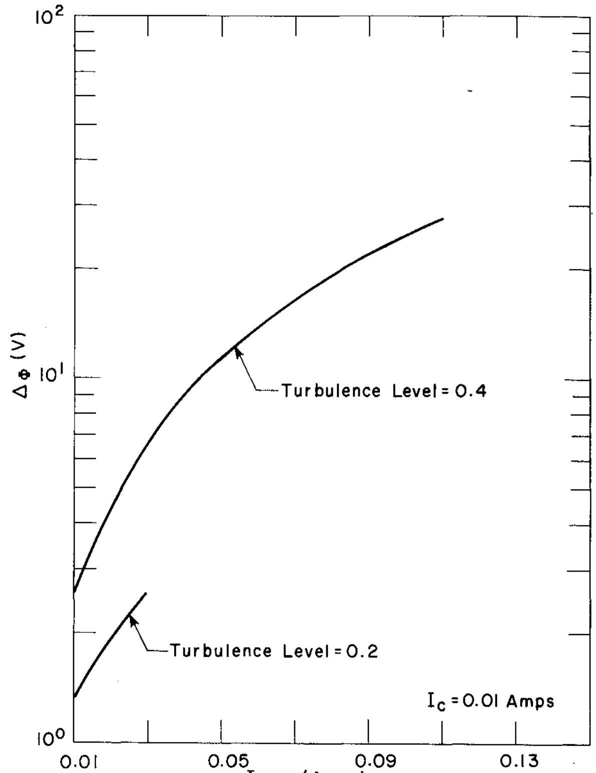

Fig. 2 Potential drop across plasma cloud vs total current.

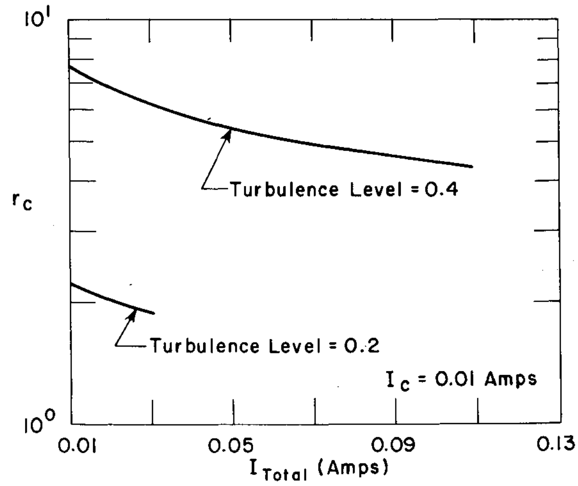

Fig. 3 Critical radius vs total current.

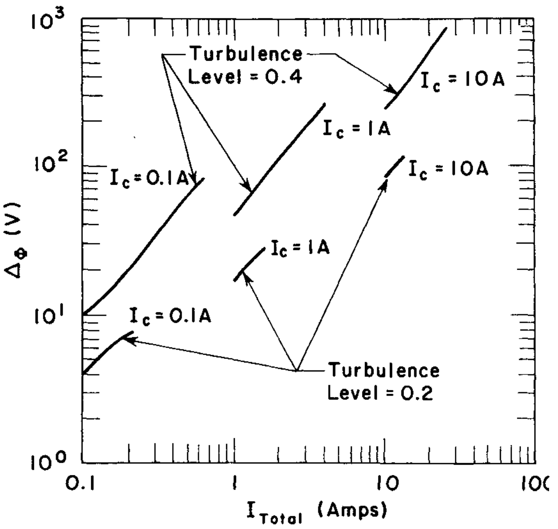

Fig. 4 Potential drop across cloud vs total current.

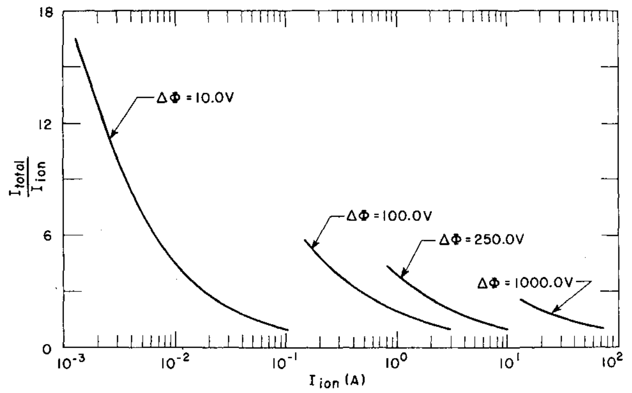

Fig. 5 Gain (I/I_c) vs ion current for several potential drops.

found that for these high currents the effects of ionization and recombination was starting to be important. This will be explored in future work when the important effects of changing the electron temperature are included. In Fig. 5 we show the gain I/I_c as a function of I_c for several potential drops. All calculations in this figure are for $\alpha = 0.4$. For small potential drops and small current levels the gain is high and close to numbers that have been observed experimentally.[18] For higher currents the gain decreases as noted previously. This suggests that in a system where large currents are desired it is better to use several contactors with low ion currents than one contactor with a large ion current. However since the plasma clouds can be meters in diameter, the plasma sources would have to be spaced many meters apart for them to function separately.

In Table 1 we show the effect of changing the plasma temperatures. This simulates the consequences of resistive heating of the plasma by the current flow. The temperature is varied by a factor of ten for a neutralized ion current of 10 A. The maximum critical radius is seen to decrease slightly with increasing temperature. This occurs because the plasma flow is taken to be always choked at the source boundary, therefore as the plasma temperature increases the initial density decreases so as to maintain a constant mass flow. In addition the classical collision frequency is strongly temperature-dependent and decreases with temperature. Therefore increasing the temperature will decrease the collision rate for the electrons and will lead to a smaller cloud size. As a consequence the total current decreases slightly. However, the voltage drop across the cloud increases slightly. This is due to the effect of the Boltzmann contribution to the potential ($\phi \sim T_e$) which would occur in the absence of any resistive contribution to the potential. Since the effect of changing the plasma temperature is relatively small we can conclude that these results are a fair approximation to a more complete model of the contactor cloud where ohmic dissipation is included. When ohmic dissipation is properly included it is likely that at high currents significant amounts of ionization will take place in the plasma cloud as the electrons stream into the collector.

IV. Conclusions

We have developed a simple model for the theory of a plasma contactor used in the ionosphere where the magnetic field is an important limiting effect. An important conclusion from this study is that large amounts of turbulence ($\nu_{\text{eff}} \sim 0.4\omega_{pe}$) are essential to effective operation of these devices. If such levels of turbulence can be achieved and sustained over tens of meters then plasma contactors can provide gains (I/I_c) of 2–6 for ion currents in the ampere range and for potential drops in the range 100–500 V. This makes them useful devices for enhancing the current flow through large electrodynamic tethers. An important subsidiary conclusion is that the gain is a decreasing function of the total current so high-ion current devices are less efficient than lower-current devices. This suggests that plasma sources with low mass flowrate and high ionization fraction may be the most effective for use in plasma contactors although the important effect of varying the ionization fraction needs to be explored more carefully.

Future work will concentrate on improving the basic model for the contactor cloud and on answering the important question of whether such high turbulence levels can be achieved over big clouds.

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